BME 575 Final Project Report

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Introduction

**Articular Cartilage**:

The articular cartilage is one of the major connective tissues of the musculoskeletal system along with tendons, ligaments, meniscus and intervertebral disc. It serves the purpose of lubricating the surface for articulation and distribute the load, enabling the wide ranges of motion in daily lives. The three major tissues of articular cartilage, meniscus, and intervertebral disc have similar mechanical properties of the tissues as they can all be considered as two immiscible phases, composed together: solid phase and fluid phase. The solid phase contains the collagen and fibrillar network and the fluid phase contains chiefly water and dissolved inorganic salts that saturates the solid matrix. Due to this composition of these two solid and fluid phases, articular cartilage can be explained by the Biphasic theory for collagenous tissues. The articular cartilage can be broken down further into its three components of collagen, proteoglycan, and H2O(water). Collagen takes up 50-73 percent of dry weight, proteoglycan takes up to 15-30% of dry weight, and H2O(water) takes up to 58-78% of wet weight.

The articular cartilage changes in composition and structure with depth from the join surface and it can be broken down into four distinct zones: superficial zone, middle/transitional zone, deep/radial zone, and zone of calcified cartilage. As an example, proteoglycan’s concentration increases from the surface to its maximum in the middle/transitional zone, and it disappears as it progresses into deep/radial zone.

Diagram

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This distinct orientation of the collagen fibrils that are showed in the diagram creates significant impact on the material properties of the articular cartilage tissue, such as tensile properties.

**Biphasic Theory**:

The KLM biphasic equations for hydrated soft tissues assume each phase to be intrinsically incompressible. Applying the assumptions, the balance of mass equation for tissues is as followed,

Diagram

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where and are the volume fractions of the solid phase and the fluid phase, and and are the velocities respectively. The volume fractions need to satisfy =1 equation, which can further drive the following equation,

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where and are the stress tensors acting on the solid phase and fluid phase. For infinitesimal strains, the KLM biphasic theory also assumes solid phase is a linearly elastic solid and the fluid phase is a Newtonian viscous fluid. We can further drive the relation between the stresses and the momentum supply as following,

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where and are Lame constants for solid phase and is the apparent viscosity of the fluid phase in cartilage, E is the infinitesimal strain tensor for the solid matrix, D is rate of deformation tensor for the interstitial fluid, and K is the coefficient of the diffusive drage caused by the relative motion between the two phases. These equations can be simplified by deriving the Navier’s equations for the solid phase and the fluid phase,

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where u is the displacement vector of the solid matrix. For infinitesimal deformation, the velocity of the solid matrix is also given by,

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These 9 equations above all together complete the biphasic theory for hydrated soft tissues with constant permeability.

For constant permeability, the coefficient of the diffusive drag K is related to permeability coefficient k by the following equation.

Diagram, schematic

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For strain-dependent permeability, the permeability function is given by the following,

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where M is material constant, and tr(E) is the dilatation of the solid matrix.

**Confined Compression Creep and Stress-relaxation**: With the articular cartilage following the biphasic model described above, when it is exposed to dynamic loading, it leads to viscoelastic behavior that is often described by stress relaxation and creep function. Creep response is seen when constant loading is applied to the specimen, as there is significant change in creep displacement in the beginning which flattens out until equilibrium is reached. This flattening rate is related to the permeability coefficient k, which can be seen in the biphasic model above. Stress relaxation is seen when the specimen goes under constant deformation, as the stress increases at constant rate and drops after the controlled displacement has been reached and flattens out until equilibrium is reached. At this equilibrium, aggregate modulus Ha can be obtained.

Objective:

The objective of this project is to obtain the permeability coefficient k and aggregate modulus Ha that are crucial to proving that the articular cartilage sample data follows the biphasic model equation. Code will be designed to obtain both k and Ha with collected data of height and diameter of the specimen, applied displacement, and load response recorded, by curve fitting the experimental data obtained from a stress-relaxation confined compression test, using the biphasic theory and stress equation.

Materials and Methods:

The specimen disc had diameter of 6.214mm and thickness of 1.104mm. Confined compression test was done, meaning that specimen displacement was only done on vertical direction.

Diagram

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The specimen disc had diameter of 6.214mm and thickness of 1.104mm. The following was the applied displacement to specimen disc.

Chart

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where h is the height of the specimen and t0=451sec.

The total stress at the surface z=h can be evaluated by the equation following, where V0=0.05h/t0

Text, schematic

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The prediction of aggregate modulus Ha and permeability coefficient k was obtained in two ways. One was Ha is determined separately from the data points from equilibrium state and k is determined by curve fitting. The other was Ha and k were both determined from curve fitting.

After the theoretical biphasic model curve fitting was completed, the quality of the curve fitting was checked by calculating the coefficient of determination r^2 using the following equation.

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Results:

The two functions were used for two different curve fitting and they were put in at the end section of the code for MATLAB code to initialize the functions before it runs the code.

Graphical user interface, application, Word

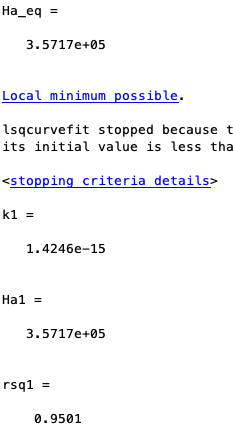
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Displacement function and obtained displacement data.

Graphical user interface, application

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Curve fitting for k. Ha obtained from equilibrium state.



Code output showing k=1.4246e-15, Ha\_eq=Ha1=3.5717e+5, r^2=0.9501. It showed r^2>0.9.

Graphical user interface

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Curve fitting for both Ha and k.

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Code outputs k=1.2993e-15, Ha=3.3048e+5, r^2=0.9817. It showed r^2>0.9.

Discussion:

Both graphs showed r^2>0.9, which indicated the strong resemblance between theoretical model curve fit with the experimental data obtained. Curve fitting method for both Ha and k showed a stronger r^2 over curve fitting for k with Ha obtained from equilibrium state. The aggregate modulus showed the scale of 10^5 and k the scale of 10^-15, which shows that the obtained values are in acceptable ranges.

References:

1. “Biphasic and Quasilinear Viscoelastic Theories for Hydrated Soft Tissues” in

“Biomechanics of Diarthrodial Joints, volume I”, by VC Mow, A Ratcliffe, and S L-Y

Woo, Springer-Verlag. (cartilage\_background.pdf on Blackboard).

1. “Structure and Function of Articular Cartilage and Meniscus” in “Basic Orthopaedic Biomechanics and Mechano-Biology” by VC Mow and R Huiskes, Lippincott William

& Wilkin.

Appendix

Code copy

%% Joon Jung BME 575 Final Project

clc

clear all

data=load('project\_data\_2020\_fall.dat');

timepoints=data(1,1);

height=data(1,2);

diameter=data(1,3);

for i=1:timepoints

time(i)=data(i+1,1);

displacement(i)=data(i+1,2);

Load(i)=data(i+1,3);

end

% mm to m conversion

height=height\*10^(-3);

diameter=diameter\*10^(-3);

displacement=displacement\*10^(-3);

t0=451;

v0=(0.05\*height)/t0;

area=(pi\*(diameter^2))/4;

% Kg to Pa conversion (Pa = (Kg\*9.8)N/m^2)

Load=(Load\*9.81)/area;

%% Displacement function

for i = 1:timepoints

t=time(i);

if t<=t0

u\_z(i) = -0.05\*height\*(t/t0);

else

u\_z(i) = -0.05\*height;

end

u\_z=abs(u\_z);

end

figure(1)

plot(time,u\_z,'b')

hold on

plot(time,displacement,'r')

xlabel('Time (s)')

ylabel('-u\_z')

title('Displacement (m)')

legend({'Applied displacement function','Displacement data'})

%% Ha calculation

Load\_eq\_avg=0;

for i=(timepoints-10):timepoints

Load\_eq\_avg=Load\_eq\_avg+(Load(i)/10);

end

stress\_equil=Load\_eq\_avg;

strain=0.05;

Ha\_eq=stress\_equil/strain

%% k Curve fitting

x0(1)=log(10^-15);

cfit=lsqcurvefit(@stressfunction1,x0,time,Load);

curvefit=stressfunction1(cfit,time);

k1=exp(cfit)

Ha1=Ha\_eq

figure(2);

plot(time,Load,'b')

hold on

plot(time,curvefit,'r')

xlabel('Time(s)')

ylabel('Force(Kg)')

title('Load response k curve fitting')

legend({'Experiment','Theoretical'})

%% r^2 for k curve fitting

Load\_avg=mean(Load);

num1=0;

den1=0;

for i=1:timepoints

num1=num1+((Load(i)-curvefit(i))^2);

den1=den1+((Load(i)-Load\_avg)^2);

end

rsq1=(1-(num1/den1))

%% Ha and k Curve fitting

x0(1)=log(10^-15);

x0(2)=log(10^5);

cfit2=lsqcurvefit(@stressfunction2,x0,time,Load);

curvefit2=stressfunction2(cfit2,time);

k2=exp(cfit2(1))

Ha2=exp(cfit2(2))

figure(3);

plot(time,Load,'b')

hold on

plot(time,curvefit2,'r')

xlabel('Time(s)')

ylabel('Force(Kg)')

title('Load response Ha and k curve fitting')

legend({'Experiment','Theoretical'})

%% r^2 for Ha and k curve fitting

num2=0;

den2=0;

for i=1:timepoints

num2=num2+((Load(i)-curvefit2(i))^2);

den2=den2+((Load(i)-Load\_avg)^2);

end

rsq2=(1-(num2/den2))

%% stress function for k

function curvefit=stressfunction1(x0,time)

data=load('project\_data\_2020\_fall.dat');

timepoints=data(1,1);

height=data(1,2);

diameter=data(1,3);

% mm to m conversion

height=height\*10^(-3);

diameter=diameter\*10^(-3);

t0=451;

v0=(0.05\*height)/t0;

area=(pi\*(diameter^2))/4;

for i=1:timepoints

time(i)=data(i+1,1);

Load(i)=data(i+1,3);

end

% Kg to Pa conversion (Pa = (Kg\*9.8)N/m^2)

Load=(Load\*9.8)/area;

% Ha calculation

Load\_avg=0;

for i=(timepoints-10):timepoints

Load\_avg=Load\_avg+(Load(i)/10);

end

stress\_equil=Load\_avg;

strain=0.05;

Ha=stress\_equil/strain;

k=exp(x0);

for i=1:timepoints

t=time(i);

sums=0;

if t<t0

for n=1:20

sums=sums+(1/(n^2))\*(1-exp((-n^2\*pi^2\*Ha\*k\*t)/height^2));

end

curvefit(i)=(-(Ha\*v0\*t)/height)-(((2\*v0\*height)/(k\*pi^2))\*(sums));

end

if t>=t0

for n=1:20

sums=sums+(1/n^2)\*(exp((-n^2\*pi^2\*Ha\*k\*(t-t0))/height^2))\*(1-exp((-n^2\*pi^2\*Ha\*k\*t0)/height^2));

end

curvefit(i)=(-(Ha\*v0\*t0)/height)-(((2\*v0\*height)/(k\*pi^2))\*(sums));

end

curvefit=abs(curvefit);

end

end

%% stress function for Ha and k

function curvefit=stressfunction2(x0,time)

data=load('project\_data\_2020\_fall.dat');

timepoints=data(1,1);

height=data(1,2);

diameter=data(1,3);

% mm to m conversion

height=height\*10^(-3);

diameter=diameter\*10^(-3);

t0=451;

v0=(0.05\*height)/t0;

area=(pi\*(diameter^2))/4;

for i=1:timepoints

time(i)=data(i+1,1);

Load(i)=data(i+1,3);

end

% Kg to Pa conversion (Pa = (Kg\*9.8)N/m^2)

Load=(Load\*9.8)/area;

k=exp(x0(1));

Ha=exp(x0(2));

for i=1:timepoints

t=time(i);

sums=0;

if t<t0

for n=1:20

sums=sums+(1/(n^2))\*(1-exp((-n^2\*pi^2\*Ha\*k\*t)/height^2));

end

curvefit(i)=(-(Ha\*v0\*t)/height)-(((2\*v0\*height)/(k\*pi^2))\*(sums));

end

if t>=t0

for n=1:20

sums=sums+(1/n^2)\*(exp((-n^2\*pi^2\*Ha\*k\*(t-t0))/height^2))\*(1-exp((-n^2\*pi^2\*Ha\*k\*t0)/height^2));

end

curvefit(i)=(-(Ha\*v0\*t0)/height)-(((2\*v0\*height)/(k\*pi^2))\*(sums));

end

curvefit=abs(curvefit);

end

end